Modeling and Forecasting of the Multi-Scale Features of Magnetospheric Dynamics during Substorms.

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Abstract. The global and multi-scale features of the solar wind-magnetosphere coupling during substorms are modeled using time series data. We introduce a new data-derived model STADY which combines a nonlinear dynamical approach with elements of statistical physics. This combined approach can be used to forecast the global or averaged features of substorms and the range of values with associated probabilities.

1. Introduction

The magnetospheric dynamics during substorms exhibits both global and multi-scale features. The global or coherent behavior of the magnetosphere is evident in a variety of large-scale processes such as plasmoid formation and ejection, field line dipolarization, global current systems, etc. At the same time a number of small-scale phenomena observed during substorms, like MHD turbulence, bursty bulk flows, current disruption, etc. are multi-scale in nature; viz. they have broad band power spectra in a wide range of perturbation scales. Such a diversity in dynamical properties seriously complicates the building a unified framework for modeling the magnetospheric dynamics during substorms.

The early data-derived models of magnetospheric substorms were inspired by the concepts of dynamical chaos. They were based on the assumption that the observed complexity of the magnetospheric dynamics is attributed to the nonlinear coupling of just a few main degrees of freedom [e.g. *Sharma*, 1995; *Klimas et al.*, 1996]. This approach led to a considerable progress in development of space weather forecasting tools based on local-linear filters and neural networks [*Prichard et al.*, 1994; *Vassiliadis et al.*, 1995; *Horton and Doxas*, 1996; *Gleisner and Lundstedt*, 1997].

However many studies have shown that not all aspects of magnetospheric dynamics during substorms conform to the hypothesis of low dimensionality and thus cannot be accounted within the framework of dynamical chaos. For example, the power spectrum of AE index data [*Tsurutani et al.*, 1990] and magnetic field fluctuations in the tail current sheet [*Ohtani et al.*, 1995] have a power law form typical for high dimensional colored noise. Moreover, detailed analyses [*Takalo et al.*, 1993, 1994] have shown that the qualitative properties of the AE time series are much more similar to bicolored noise than to low-dimensional chaotic systems.

One of the models used to explain the multi-scale properties of the magnetospheric dynamics is self-organized criticality (SOC) [*Bak et al.*, 1987]. A system in SOC is modeled by a sand-pile or other non-equilibrium cellular automata which evolve to a steady-state critical point due to the fine tuning of the control parameters [*Vespignani and Zapperi*, 1998]. In the vicinity of a critical point the energy transport in the system is carried out by avalanches whose sizes are distributed according to a power law. This feature of SOC has been exploited to account for the power law spectra observed in the magnetosphere during substorms [e.g. *Consolini, 1997*; *Chapman et al.*, 1998; *Takalo et al.*, 1999; *Klimas et al.*, 2000]. However, the fine tuning of parameters required to approach the criticality corresponds to the vanishing values of the input parameter [*Vespignani and Zapperi*, 1998]. This makes the system effectively autonomous and thus questions the validity of SOC to the magnetosphere, whose dynamics is to a large extent driven by the solar wind input. Moreover, SOC models generally cannot account for the large-scale coherent features of the magnetosphere since in most SOC models the multi-scale properties of the system are essentially independent of the global dynamics. The only exception to this was observed in a sand-pile model [*Chapman et al.*, 1998] in which scale-invariant avalanches were found to coexist with system-wide large-scale events.

Our goal is to reconcile the global and multi-scale features of solar wind-magnetosphere coupling during substorms. For this purpose we combine nonlinear dynamical approach with elements of statistical physics. The methods of nonlinear dynamics are used to reconstruct the phase space of the system and to forecast the global constituent of the magnetospheric dynamics using local-linear filters (LLF). However, it has been shown recently that LLF models leave out a significant portion of the time-series and these features correspond to the multi-scale component of the dynamics [*Ukhorskiy et al.*, 2002a]. This multi-scale constituent of the magnetospheric dynamics is high dimensional, similar to the colored noise, and does not allow deterministic predictions, thus imposing limitations on the predicting ability of the dynamical models.

The magnetospheric dynamics during substorms shares a number of properties with non-equilibrium phase transitions [*Sitnov et al.*, 2000; *Sitnov et al.*, 2002]. In particular, using global singular spectrum analysis of VB_S-AL data *Sitnov et al.* [2000] have shown that the global magnetospheric dynamics is organized in a manner similar to the "pressure-temperature-density" diagram of the water-steam system. They also established the relation between the magnitude of the largest fluctuations of AL time derivative and the solar wind parameters similar to the β input-output critical exponent [*Sitnov et al.*, 2001]. The subsequent analyses [*Ukhorskiy et al.*, 2002a] have shown that LLF dynamical models are very similar to the mean-field approach in phase transitions since its output is obtained by an averaging over a chosen range of scales in the reconstructed input-output phase space. Thus, the multi-scale features of the time series not captured by LLFs are essentially the deviations of the data from the mean-field model. According to the phase transition analogy the magnitudes of these fluctuations should be related to the solar wind input in a probabilistic fashion similar to the input-output critical exponent. In this paper we demonstrate that such a relation can be established in terms of conditional probability and this result can be used to improve current forecasting tools. In the following we present the first results obtained with use of a new model STADY that combines the STAtistical and DYnamical features of the magnetosphere. The model output consists of a dynamical prediction from a low-dimensional model and an estimate of its deviations from the data, computed from the conditional probability of the events.

2. Solar Wind – Magnetosphere Data Base

The data-derived model STADY is built using the correlated database of solar wind and geomagnetic time series compiled by *Bargatze et al.* [1985], which consists of solar wind data from IMP 8 spacecraft and the corresponding values of the auroral indices with 2.5 min resolution. The database consists of 34 isolated intervals, and contain 42,216 points total. Each interval represents isolated auroral activity preceded and followed by at least two-hour-long quite periods ($VB_s \approx 0$, AL<50 nT), and are arranged in the order of increasing geomagnetic activity. The solar wind convective electric field VB_S is taken as the input in the model. The magnetospheric response to the solar wind activity is represented by the AL index and is the output of the model. To facilitate the use of VB_S and AL data in a joint input-output phase space, the data sets are normalized by their respective standard deviations.

3. Dynamical Model

The main concept underlying the dynamical approach is the time-delay embedding technique which was initially developed for autonomous chaotic systems [see review: *Abarbanel et al.,* 1993]. It is assumed that a scalar time series data of an observable quantity is a function only of the state of the underlying system and contain all the information necessary to determine its dynamics. Thus, if a space large enough to unfold the dynamical attractor is reconstructed from the time series and the present state of the system is identified, then the information about the future can be inferred from the known evolution of similar states.

For driven dynamical systems the states of the system in the reconstructed phase space are given by the input-output delay vectors:

$$\left(\vec{\mathbf{I}}_n, \vec{\mathbf{O}}_n\right)^T = \left(\mathbf{I}_n, ..., \mathbf{I}_{n-(M-1)}, \mathbf{O}_{n-1}, ..., \mathbf{O}_{n-(M-1)}\right) \quad (1)$$

where $\mathbf{I(t_n) = I_n}$ and $\mathbf{O(t_n) = O_n}$ are the input and the output time series, and M is the total number of delays. A proper reconstruction requires a time delay that is long enough to unfold the underlying dynamics and short enough so that relevant features are not lost. Once such a reconstruction is achieved, the value of the output at the next time step is related to the current state of the system by some nonlinear map **F**:

$$\mathbf{O}_{n+1} = \mathbf{F}(\vec{\mathbf{I}}_n, \vec{\mathbf{O}}_n) \quad (2)$$

The linear expansion of **F** gives the well known expression for the local-liner ARMA filters:

$$\mathbf{O}_{n+1} = \sum_{i=-1}^{M-1} \alpha_i \cdot \mathbf{I}_{n-i} + \sum_{j=Tprd/dt}^{M-1} \beta_j \cdot \mathbf{O}_{n-j} \quad (3)$$

The filter coefficients α_i and β_j are calculated using the known data, which is referred to as the training set. The training set is searched for the states similar to the current, that is the states that are closest to it, as measured by the distance in the embedding space defined using the Euclidean metrics. These states are referred to as the nearest neighbors.

To model the solar wind-magnetosphere coupling during substorms we use local-linear filters (LLF) defined by (3) with $\mathbf{I=VB_S(t)/\sigma_{VBs}}$ and $\mathbf{O=AL(t)/\sigma_{AL}}$. An example of 1 hour predictions of AL for a high activity period (interval 32 of *Bargatze et al.* [1985] data set) obtained by this model is shown in Fig 1. It can be seen from the figure that LLF reproduces the long-term global variations in data well and this feature is now used to forecast the average level of substorm activity. This component of AL is regular and low-dimensional by construction. Thus, the portion of the time series reproduced by LLF corresponds to the global and coherent features of the magnetospheric dynamics. However, it is also evident from the plot that LLF fails to yield the abrupt variations and peaks. This feature of LLF is generic to the magnetosphere and does not depend on the level of substorm activity [*Ukhorskiy et al.*, 2002a]. Also the inability of LLF to reproduce the sharp variations in data is due to the high dimensionality of the multi-scale constituents of AL time series. The properties of that part of AL not captured by LLF dynamical model are very different from the multi-scale properties of low-dimensional chaotic systems like Lorenz attractor or Mackey-Glass system, in which the scale-invariance is reconciled with low

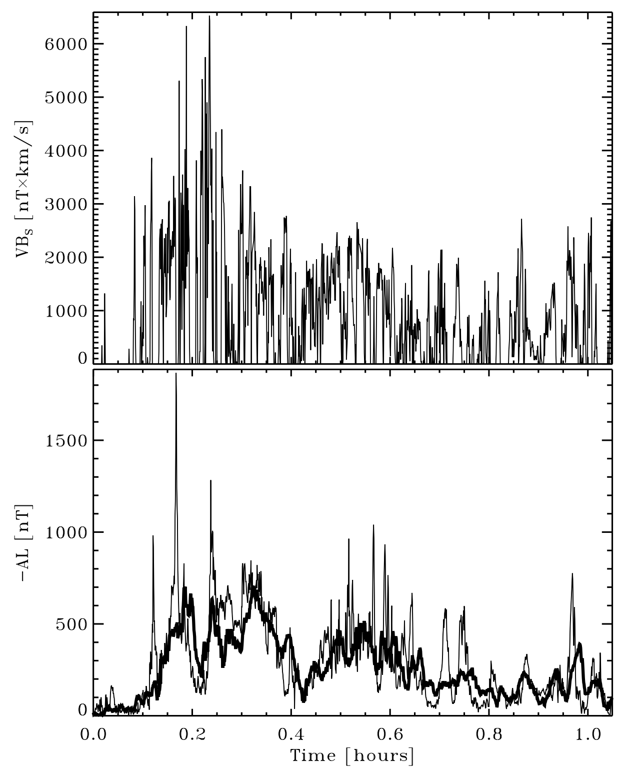

Figure 1. LLF 1 hour predictions of the high activity Bargatze 32nd interval. The solar wind input data (VB_S) is shown on the upper panel. AL time series together with the model output (bold line) are shown on the bottom panel.

dimensionality due to the fractal structure of their attractors. Both statistical and dynamical properties of this component of AL are closer to those of colored noise.

The analyses of the delay embedding of AL-VB_S time series [*Ukhorskiy et al.*, 2002b] have shown that the averaging over a large number (NN) of nearest neighbors carried out in the embedding space smoothes away the small-scale high dimensional dynamical constituents. This averaging is what yields the more regular low dimensional component from the time series. The higher the value of NN the wider is the range of scales which are smoothed away and the smaller is the effective dimensionality of the averaged system. It was also found that a single set of parameters (M, NN) corresponding to the reconstruction of the "correct" dynamical system does not exist. The time delay embedding should be seen rather as a process of striking a balance between the level of "noise" (the range of scales over which the averaging is performed) and the complexity (the effective dimensionality of the averaged system) in a "noisy" dynamical system. Similar conclusions were reached by *Stark* [2001] from the theoretical analysis of time delay embedding in stochastic dynamical systems.

The averaging that stabilizes the filter output and helps in extracting the coherent component from AL data is also the cause of information loss, leading to significant limitations on our ability to predict by using the dynamical approach. In particular, due to the averaging the sharpest peaks in data always come out strongly smoothed, and the actual magnitudes of substorms are not predicted. Thus, to improve the accuracy of the data-derived forecasting tools a new approach beyond the dynamical modeling is required.

4. Probabilistic Approach

The dynamical model output is constructed by taking phase space averages of a number of states similar to the current state of the system, viz. the nearest neighbors. The reconstructed phase space is divided into clusters and this yields the probability measure of the events constituting that cluster. The size of the cluster is defined by the radius of a sphere containing the set of nearest neighbors. To obtain the model output the average is taken only over the states within a given cluster while the states outside this cluster are considered to be independent and therefore do not contribute to the output. Thus, the LLF is similar to the mean-field approach in thermodynamics, and this recognition can be used to build a link between the two components based on dynamics and statistical physics. The output of the dynamical model corresponds to the trajectories in a truncated reconstructed phase space that lie on a low-dimensional surface defined in the mean-field sense. The differences between the model output and the real AL data correspond to the deviations of real trajectories from this mean-field surface due to the high dimensionality of the multi-scale portion of AL and due to the dynamic nature of substorms. In the SOC approach these fluctuations are considered autonomous and thus should not depend on the solar wind input. On the other hand, in the models based on phase transition there should be a statistical relationship between the multi-scale fluctuations and the solar wind features. Such a relationship was obtained and an input-output critical exponent computed from the Bargatze et al. [1985] data set by *Sitnov et al.* [2001]. In this case the probability distribution of substorms is a function of solar wind parameters and should be defined in terms of a conditional probability $\mathbf{P(AL|\,solar\ wind)}$. The output of the dynamical model (AL_{ARMA}) is to a large extent defined by the solar wind parameters as

$$\mathbf{AL_{ARMA} \approx \int_0^{\infty} VB_S(t-\tau)\cdot f(\tau)d\tau + \int_{T_{prd}}^{\infty} AL(t-\tau)\cdot g(\tau)d\tau} \quad (4)$$

where T_{prd} is the prediction time scale. Therefore AL_{ARMA} can be used as a measure of the solar wind input, and the conditional probability of AL fluctuations can be calculated in the form of $\mathbf{P(AL|\,AL_{ARMA})}$. As will be shown later the advantage of this particular form of the distribution function is that it can be used directly to improve the forecasting of AL. The probability distribution function $\mathbf{P(AL,\,AL_{ARMA})}$ is shown on Fig 2. The straight line AL=AL_{ARMA} corresponds to the mean-field surface, i.e. the output of dynamical model. The distribution of AL about a given value of AL_{ARMA} sets the conditional probability $\mathbf{P(AL|\,AL_{ARMA})}$. As can be seen from the plot there is a clear dependence of

P(AL|AL$_{ARMA}$) on AL$_{ARMA}$ this quantifies the relation between the solar wind input and the fluctuations of AL about the mean-field surface.

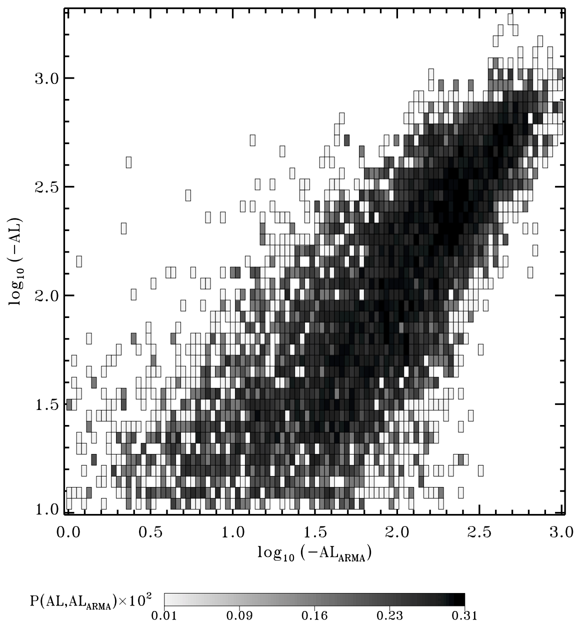

Figure 2. **P(AL, AL$_{ARMA}$)** probability distribution function calculated using Bargatze database.

5. STADY

With the set of **P(AL|AL$_{ARMA}$)** calculated using the training set we can now proceed with the construction of the new model STADY which combines the dynamical model (DY) with statistical approach (STA). To predict the value of AL at the (n+1) time step we first calculate AL$_{ARMA}$(t_{n+1}) using the dynamical model (3) and the known history of AL for $t<(t_{n+1}-T_{prd})$ and VB$_S$ for $t\leq t_{n+1}$. As discussed above AL$_{ARMA}$(t_{n+1}) should be considered as an estimate of only the average level of the substorm activity since the dynamical model often underestimates the AL peaks. This is where the statistical part of the model comes to the play. Knowing AL$_{ARMA}$(t_{n+1}) we can estimate the magnitude of AL deviation from the output of the deterministic model with use of the distribution function **P(AL|AL$_{ARMA}$(t_{n+1}))**. This function not only specifies the largest possible value of AL for a given AL$_{ARMA}$, but also ranks it in terms of the probability. An example of AL predictions with use of STADY is shown in Fig 3 for the data interval shown in Fig 1. As can be seen from the plot STADY yields not only the long term deterministic predictions of the global features of the time series but also yields estimates of the high dimensional multi-scale component of AL in a probabilistic fashion. Thus, for given solar wind conditions STADY can forecast the magnitudes of the substorms and their their associated probabilities, and thus can be used as a practical forecasting tool.

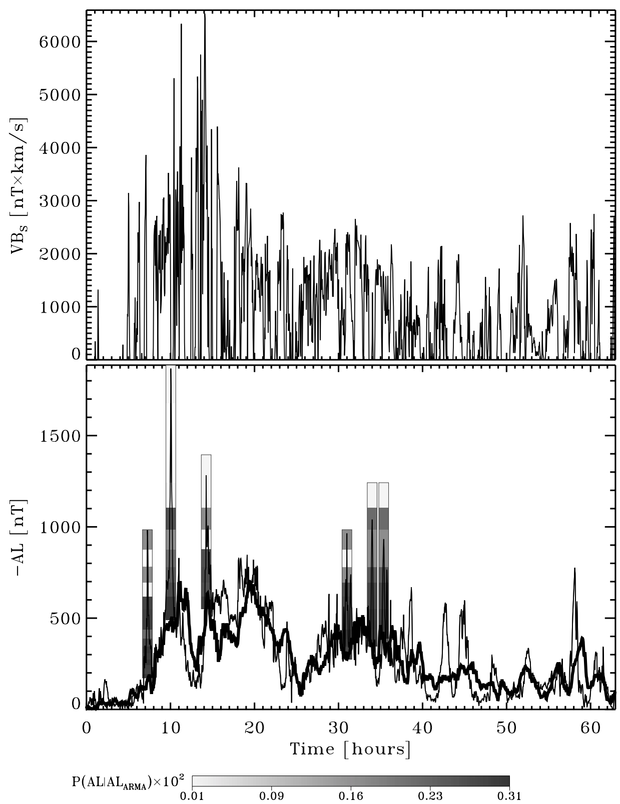

Figure 3. STADY predictions of the high activity Bargatze 32nd interval. The solar wind input data (VB$_S$) is shown on the upper panel. AL time series are shown on the bottom panel. The dynamical model output is indicated by the bold line. Color bars show the probability of the largest deviations (AL>900 nT) of real data from the dynamical model output calculated using **P(AL|AL$_{ARMA}$).**

6. Conclusions

A new approach for data-derived modeling of the solar wind-magnetosphere coupling during substorms is introduced. This model, STADY, combines the nonlinear dynamical approach with elements of statistical physics, leading to a reconciliation of the global and multi-scale aspects of the magnetosphere during substorms. The dynamical part of the model leads to the deterministic predictions of the globally coherent components of the time series, while the statistical part yields probabilistic predictions of the multi-scale constituents. The combined STADY approach leads to a significant improvement in the space weather forecasting tools since it yields not only the deterministic predictions of the average level of the magnetospheric activity but also the probabilities of the range of geomagnetic activity for given solar wind conditions.

7. References

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